S602. $A = \frac{1}{n+1}\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3}+\frac{1}{5}\right)\ldots\left(1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}\right)$
and
$B = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{4}\right)\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)\ldots\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2n}\right)$
Compare A and B.

Proposed by Nguyen Viet Hung.

Solution by Arkady Alt, San Jose, California, USA.

Let $H_n = 1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}, S_n := 1+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n-1}$,
$T_n := \frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2n} = \frac{1}{2}H_n$, and note that $S_n + T_n = H_{2n},\ n \in \mathbb{N}$
Then $A_n = \frac{1}{n+1}\prod_{k=2}^{n} S_k$ and $B_n = \frac{1}{2}\prod_{k=2}^{n} T_k, n \in \mathbb{N}\diagdown\{1\}$.
We have $A_2 = \frac{1}{2+1}\left(1+\frac{1}{3}\right) = \frac{4}{9},\ B_2 = \frac{1}{2}\left(\frac{1}{2}+\frac{1}{4}\right) = \frac{3}{8}$ and, therefore, $A_2 > B_2$
and we will prove that $\frac{A_{n+1}}{A_n} > \frac{B_{n+1}}{B_n}$ for any $n \in \mathbb{N}\diagdown\{1\}$.

Indeed, $\frac{A_{n+1}}{A_n} > \frac{B_{n+1}}{B_n} \Leftrightarrow \dfrac{\frac{1}{n+2}\prod_{k=2}^{n+1} S_k}{\frac{1}{n+1}\prod_{k=2}^{n} S_k} > \dfrac{\prod_{k=2}^{n+1} T_k}{\prod_{k=2}^{n} T_k} \Leftrightarrow \frac{S_{n+1}(n+1)}{n+2} > T_{n+1} \Leftrightarrow$
$S_{n+1} > \left(1+\frac{1}{n+1}\right)T_{n+1}, n \in \mathbb{N}\diagdown\{1\} \Leftrightarrow S_n > \left(1+\frac{1}{n}\right)T_n, n \in \mathbb{N}\diagdown\{1,2\}$
and we have $S_n > \left(1+\frac{1}{n}\right)T_n \Leftrightarrow S_n + T_n > \left(2+\frac{1}{n}\right)T_n \Leftrightarrow$
$H_{2n} > \frac{1}{2}\left(2+\frac{1}{n}\right)H_n = \left(1+\frac{1}{2n}\right)H_n \Leftrightarrow H_{2n} - H_n > \frac{1}{2n}H_n \Leftrightarrow$
$\sum_{k=1}^{n}\frac{1}{n+k} > \sum_{k=1}^{n}\frac{1}{2n\cdot k} \Leftrightarrow \sum_{k=1}^{n}\left(\frac{1}{n+k} - \frac{1}{2n\cdot k}\right) = \sum_{k=1}^{n}\frac{k(2n-1)-n}{2kn(k+n)} > 0,$
where the latter inequality holds because for $k \in \{1,\ldots,n\}$ and $n > 2$
we have $k(2n-1)-n \geq 1\cdot(2n-1)-n \geq 1\cdot(2n-1)-n = n-1 > 0.$
Since $A_2 > B_2$ and $\frac{A_{n+1}}{A_n} > \frac{B_{n+1}}{B_n}$ for any $n \in \mathbb{N}\diagdown\{1\}$ then by MI
$A_n > B_n$ for any $n \in \mathbb{N}\diagdown\{1\}$.